

# GED Mathematical Reasoning — Table of Contents

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# 1. Quantitative Problem Solving

Quantitative problem solving involves the ability to use numerical and mathematical concepts to solve practical problems. It is a core skill in mathematics reasoning, and mastering it requires a strong understanding of number operations, number sense, and how numbers behave in different contexts.

## 1.1 Number Operations and Number Sense

Number sense refers to an intuitive understanding of numbers, their magnitude, relationships, and how they are affected by operations. Number operations are the procedures used to manipulate numbers according to mathematical rules.

### 1.1.1 Integers, Fractions, and Decimals

**Integers** are whole numbers and their negatives, including zero:

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

They can be positive (e.g., 5), negative (e.g.,  $-5$ ), or zero.

**Fractions** represent parts of a whole and are expressed as:

$$\frac{a}{b}, \quad b \neq 0$$

where  $a$  is the numerator and  $b$  is the denominator. Fractions can be proper ( $\frac{3}{4}$ ), improper ( $\frac{7}{4}$ ), or mixed numbers ( $1\frac{3}{4}$ ).

**Decimals** are another way to represent fractions using powers of ten:

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

They are useful for precise calculations in science, engineering, and everyday contexts.

Example: Convert 1.25 to a fraction:

$$1.25 = 1 + 0.25 = 1 + \frac{25}{100} = 1 + \frac{1}{4} = \frac{5}{4}$$

### 1.1.2 Place Value and Rounding

**Place value** tells us the value of a digit based on its position. In the number 4,572.39:

Thousands: 4,   Hundreds: 5,   Tens: 7,   Ones: 2,   Tenths: 3,   Hundredths: 9

**Rounding** is the process of approximating a number to a specific place value.

- Round 3,746 to the nearest hundred: look at the tens digit (4), which is less than 5, so it rounds to 3,700.
- Round 5.678 to two decimal places: look at the third decimal (8), which is greater than 5, so it rounds to 5.68.

### 1.1.3 Factors and Multiples

**Factors** of a number are integers that divide it exactly without a remainder. For example:

$$\text{Factors of } 12 = \{1, 2, 3, 4, 6, 12\}$$

**Multiples** of a number are obtained by multiplying it by integers:

$$\text{Multiples of } 5 = \{5, 10, 15, 20, \dots\}$$

**Prime factors** are factors that are prime numbers. Using prime factorization:

$$60 = 2^2 \times 3 \times 5$$

### 1.1.4 Ratios, Rates, and Proportions

A **ratio** compares two quantities:

$$\text{Ratio of apples to oranges} = \frac{\text{apples}}{\text{oranges}} = 3 : 2$$

A **rate** is a ratio that compares quantities of different units:

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{120 \text{ km}}{2 \text{ h}} = 60 \text{ km/h}$$

A **proportion** states that two ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \text{or} \quad a : b = c : d$$

Example: If 3 pencils cost \$1.50, how much do 10 pencils cost?

$$\frac{3}{1.50} = \frac{10}{x} \quad \Rightarrow \quad x = \frac{10 \times 1.50}{3} = 5$$

So, 10 pencils cost \$5.

### 1.1.5 Percents

A **percent** is a ratio comparing a number to 100:

$$\text{Percent} = \frac{\text{part}}{\text{whole}} \times 100\%$$

**Percent increase:**

$$\text{Percent Increase} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

**Percent decrease:**

$$\text{Percent Decrease} = \frac{\text{original value} - \text{new value}}{\text{original value}} \times 100\%$$

Example: A jacket originally costs \$80 and is now on sale for \$60.

$$\text{Percent Decrease} = \frac{80 - 60}{80} \times 100\% = 25\%$$

### 1.1.6 Scientific Notation

**Scientific notation** expresses very large or small numbers as:

$$a \times 10^n \quad \text{where} \quad 1 \leq |a| < 10 \text{ and } n \text{ is an integer.}$$

Example:

$$4500 = 4.5 \times 10^3, \quad 0.00032 = 3.2 \times 10^{-4}$$

**Operations in scientific notation:**

- Multiplication:  $(a \times 10^m) \times (b \times 10^n) = (a \times b) \times 10^{m+n}$
- Division:  $\frac{a \times 10^m}{b \times 10^n} = \left(\frac{a}{b}\right) \times 10^{m-n}$

Example:

$$(2 \times 10^4) \times (3 \times 10^5) = 6 \times 10^9$$

# Measurement and Data

## 1 Measurement and Data

Measurement and data concepts are essential for quantitative problem-solving. This section covers measurement systems, geometric measurements, unit conversions, interpreting data, statistical measures, and simple probability.

### 1.1 Units of Measurement (U.S. Customary and Metric)

Measurement units provide a standard way to describe quantities. The two most common systems are:

- **U.S. Customary System** – Used mainly in the United States; includes units such as inches, feet, yards, miles (length), ounces, pounds, tons (mass/weight), and cups, pints, quarts, gallons (volume).
- **Metric System (SI Units)** – Used internationally; includes meters (length), liters (volume), and grams (mass), with prefixes such as milli-, centi-, kilo- for scaling.

#### Examples:

- Length: 1 foot = 12 inches, 1 meter = 100 centimeters.
- Mass: 1 kilogram = 1000 grams.
- Volume: 1 liter = 1000 milliliters.

Understanding both systems and converting between them is important in global contexts.

### 1.2 Perimeter, Area, Surface Area, and Volume

**Perimeter:** The total length around a two-dimensional shape.

$$P_{\text{rectangle}} = 2(l + w)$$

$$P_{\text{triangle}} = a + b + c$$

**Area:** The measure of space inside a two-dimensional shape.

$$A_{\text{rectangle}} = l \times w$$

$$A_{\text{triangle}} = \frac{1}{2}b \times h$$

$$A_{\text{circle}} = \pi r^2$$

**Surface Area:** The total area covering the outside of a three-dimensional object.

$$SA_{\text{cube}} = 6s^2$$

$$SA_{\text{rectangular prism}} = 2(lw + lh + wh)$$

$$SA_{\text{sphere}} = 4\pi r^2$$

**Volume:** The space occupied by a three-dimensional object.

$$V_{\text{cube}} = s^3$$

$$V_{\text{rectangular prism}} = l \times w \times h$$

$$V_{\text{cylinder}} = \pi r^2 h$$

## 1.3 Converting Between Units

**Within the same system:**

$$1 \text{ yard} = 3 \text{ feet} \quad 1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ kilometer} = 1000 \text{ meters} \quad 1 \text{ meter} = 100 \text{ centimeters}$$

**Between systems:**

$$1 \text{ inch} \approx 2.54 \text{ centimeters}$$

$$1 \text{ pound} \approx 0.4536 \text{ kilograms}$$

$$1 \text{ gallon} \approx 3.785 \text{ liters}$$

Unit conversion often uses **conversion factors**, expressed as fractions equal to 1, to multiply or divide given quantities.

## 1.4 Reading and Interpreting Graphs and Tables

Data can be presented visually to make patterns easier to see. Common representations include:

- **Bar Graphs** – Compare quantities across categories.
- **Line Graphs** – Show changes over time.
- **Pie Charts** – Show proportions of a whole.
- **Tables** – Organize raw numerical data for reference.



**Example:** In a temperature vs. time line graph, the slope of the line indicates the rate of temperature change. In a pie chart of monthly expenses, each slice's angle corresponds to a percentage of the total.

When interpreting graphs, always:

- Read the **title** for context.
- Check the **axes labels** for units and variables.
- Note the **scale** to interpret values correctly.

## 1.5 Mean, Median, Mode, and Range

These are basic statistical measures that summarize data:

- **Mean** (Average):

$$\text{Mean} = \frac{\text{Sum of all data points}}{\text{Number of data points}}$$

- **Median:** The middle value when the data is ordered; if there is an even number of values, take the average of the two middle ones.
- **Mode:** The most frequently occurring value(s) in the data set.
- **Range:** The difference between the largest and smallest values.

$$\text{Range} = \text{Maximum} - \text{Minimum}$$

**Example:** For the data set  $\{4, 6, 8, 8, 10\}$ :

- Mean =  $(4 + 6 + 8 + 8 + 10)/5 = 7.2$
- Median = 8
- Mode = 8
- Range =  $10 - 4 = 6$

## 1.6 Simple Probability

Probability measures the likelihood of an event occurring.

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

**Example:** Rolling a fair 6-sided die:

$$P(\text{rolling a 4}) = \frac{1}{6}$$

**Example:** Drawing a red card from a standard deck:

$$P(\text{red card}) = \frac{26}{52} = \frac{1}{2}$$

Probabilities are always between 0 (impossible event) and 1 (certain event). They can also be expressed as percentages.

**Complementary Events:** If  $P(E)$  is the probability of event  $E$ , then:

$$P(\text{not } E) = 1 - P(E)$$

**Compound Events:**

- **Independent events:**  $P(A \text{ and } B) = P(A) \times P(B)$
- **Dependent events:** Probability changes based on previous outcomes.

Understanding probability helps in making predictions and informed decisions.

# Basic Geometry

## 1 Basic Geometry

Basic geometry is essential for understanding shapes, sizes, and spatial reasoning. This section covers lines, angles, triangles, polygons, circles, the Pythagorean theorem, and transformations.

### 1.1 Lines, Angles, and Triangles

**Lines:** A line extends infinitely in both directions and has no thickness. **Line segments** are parts of a line with two endpoints.

**Angles:** Formed where two lines meet at a vertex.

- Acute angle:  $< 90^\circ$
- Right angle:  $= 90^\circ$
- Obtuse angle:  $> 90^\circ$  and  $< 180^\circ$
- Straight angle:  $= 180^\circ$

**Triangles:** A polygon with three sides. **Types:**

- Equilateral: all sides and angles equal
- Isosceles: two sides equal, two angles equal
- Scalene: all sides and angles different
- Right: one angle  $90^\circ$

**Triangle Properties:**

Sum of interior angles  $= 180^\circ$

## 1.2 Properties of Quadrilaterals and Polygons

**Quadrilaterals:** Four-sided polygons with the sum of interior angles  $360^\circ$ . Examples:

- Square: all sides equal, all angles  $90^\circ$
- Rectangle: opposite sides equal, all angles  $90^\circ$
- Parallelogram: opposite sides equal and parallel, opposite angles equal
- Rhombus: all sides equal, opposite angles equal
- Trapezoid: one pair of parallel sides

**Polygons:** Closed figures with three or more sides. **Sum of interior angles:**

$$\text{Sum} = (n - 2) \times 180^\circ, \quad n = \text{number of sides}$$

## 1.3 Circles: Radius, Diameter, and Circumference

**Radius:** Distance from center to any point on the circle, denoted  $r$ .

**Diameter:** Distance across the circle through the center,  $d = 2r$ .

**Circumference:** Distance around the circle,

$$C = 2\pi r = \pi d$$

**Area:** Space inside the circle,

$$A = \pi r^2$$

## 1.4 Pythagorean Theorem

For a right triangle with legs  $a$ ,  $b$  and hypotenuse  $c$ :

$$a^2 + b^2 = c^2$$

**Example:** A triangle with legs 3 and 4:

$$c^2 = 3^2 + 4^2 = 9 + 16 = 25 \quad \Rightarrow \quad c = 5$$

## 1.5 Transformations (Translations, Rotations, Reflections, Dilations)

- **Translation:** Moving a figure without rotation or resizing.
- **Rotation:** Turning a figure around a fixed point by a given angle.
- **Reflection:** Flipping a figure over a line (mirror image).
- **Dilation:** Resizing a figure larger or smaller, proportional scaling.

**Example:**

- Translation by vector  $(3, 2)$  moves point  $(x, y)$  to  $(x + 3, y + 2)$ .
- Rotation  $90^\circ$  counterclockwise around origin:  $(x, y) \rightarrow (-y, x)$ .
- Reflection across  $x$ -axis:  $(x, y) \rightarrow (x, -y)$ .
- Dilation with scale factor  $k$ :  $(x, y) \rightarrow (kx, ky)$ .

# Algebraic Problem Solving: Expressions and Polynomials

## 1 Expressions and Polynomials

Algebraic expressions form the foundation of algebraic problem solving. This section covers variables and constants, evaluating expressions, combining like terms, and factoring polynomials.

### 1.1 Variables and Constants

**Variables:** Symbols (usually letters) that represent unknown or changing quantities. Example: In  $x + 5 = 12$ ,  $x$  is a variable.

**Constants:** Fixed numerical values. Example: In  $3x + 7$ , 7 is a constant.

**Coefficients:** Numbers multiplying the variables. Example: In  $5y^2 + 3y + 7$ , 5 is the coefficient of  $y^2$ , 3 is the coefficient of  $y$ .

### 1.2 Evaluating Expressions

To evaluate an expression, substitute numbers for the variables and perform the operations according to the order of operations (PEMDAS).

**Example:** Evaluate  $3x^2 - 2x + 5$  for  $x = 2$ .

$$3(2)^2 - 2(2) + 5 = 3(4) - 4 + 5 = 12 - 4 + 5 = 13$$

### 1.3 Combining Like Terms

Like terms are terms that have the same variable raised to the same power. Combine them by adding or subtracting their coefficients.

**Example:** Simplify  $5x + 3 - 2x + 7$

$$(5x - 2x) + (3 + 7) = 3x + 10$$

**Example:** Simplify  $4y^2 + 3y - 2y^2 + 7y + 1$

$$(4y^2 - 2y^2) + (3y + 7y) + 1 = 2y^2 + 10y + 1$$

## 1.4 Factoring Polynomials

Factoring is the process of expressing a polynomial as a product of simpler polynomials. Common methods include:

### 1. Factoring out the Greatest Common Factor (GCF):

$$6x^3 + 9x^2 = 3x^2(2x + 3)$$

**2. Factoring Trinomials:** For  $ax^2 + bx + c$ , find two numbers that multiply to  $ac$  and add to  $b$ .

**Example:** Factor  $x^2 + 5x + 6$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

### 3. Factoring Difference of Squares:

$$a^2 - b^2 = (a - b)(a + b)$$

**Example:** Factor  $x^2 - 9$

$$x^2 - 9 = (x - 3)(x + 3)$$

### 4. Factoring by Grouping:

$$x^3 + 3x^2 + 2x + 6$$

Group terms:  $(x^3 + 3x^2) + (2x + 6) = x^2(x + 3) + 2(x + 3)$  Factor out common binomial:  
 $(x^2 + 2)(x + 3)$

## 1.5 Practice Problems

1. Simplify:  $7x + 3 - 4x + 8$
2. Evaluate:  $2x^2 + 5x - 3$  for  $x = -1$
3. Factor:  $x^2 - 16$
4. Factor:  $3x^2 + 12x$

# Algebraic Problem Solving: Equations and Inequalities

## 1 Equations and Inequalities

Equations and inequalities are fundamental in algebra, used to represent relationships between quantities and to solve problems.

### 1.1 Solving Linear Equations

A **linear equation** is an equation of the form  $ax + b = 0$ , where  $a \neq 0$ . The goal is to find the value of the variable that makes the equation true.

**Steps:**

1. Simplify both sides by combining like terms.
2. Move variable terms to one side and constants to the other.
3. Solve for the variable.

**Example:** Solve  $3x + 5 = 14$

$$3x = 14 - 5$$

$$3x = 9$$

$$x = 3$$

### 1.2 Solving Linear Inequalities

A **linear inequality** is similar to a linear equation but uses inequality symbols:  $<$ ,  $\leq$ ,  $>$ ,  $\geq$ .

**Key rule:** When multiplying or dividing both sides by a negative number, reverse the inequality sign.

**Example:** Solve  $2x - 5 > 3$

$$2x > 8$$

$$x > 4$$



**Graphical representation:** On a number line,  $x > 4$  is shown as an open circle at 4 with a shaded line to the right.

### 1.3 Systems of Linear Equations

A **system of linear equations** consists of two or more linear equations with the same variables. The solution is the point(s) where the equations intersect.

**Methods of solving:**

- **Graphical method:** Plot both lines and find the intersection.
- **Substitution method:** Solve one equation for a variable and substitute into the other.
- **Elimination method:** Add or subtract equations to eliminate a variable.

**Example:** Solve the system

$$\begin{cases} x + y = 7 \\ 2x - y = 3 \end{cases}$$

**Solution (Substitution method):** From the first equation:  $y = 7 - x$  Substitute into the second:  $2x - (7 - x) = 3$

$$2x - 7 + x = 3$$

$$3x = 10$$

$$x = \frac{10}{3}, \quad y = 7 - \frac{10}{3} = \frac{11}{3}$$

### 1.4 Word Problems with Equations

Linear equations and systems can model real-world situations. The process involves:

1. Defining variables for unknown quantities.
2. Writing equations based on relationships described in words.
3. Solving the equations and interpreting the solution.

**Example:** A store sells pens for \$2 each and notebooks for \$3 each. A customer buys a total of 10 items for \$25. How many pens and notebooks did the customer buy?

**Solution:** Let  $x$  = number of pens,  $y$  = number of notebooks.

$$x + y = 10$$

$$2x + 3y = 25$$

Use substitution:  $y = 10 - x$ , then

$$2x + 3(10 - x) = 25$$

$$2x + 30 - 3x = 25$$

$$-x + 30 = 25$$

$$x = 5, \quad y = 5$$

The customer bought 5 pens and 5 notebooks.

## 1.5 Practice Problems

1. Solve:  $5x - 7 = 18$

2. Solve:  $-3x + 4 \leq 10$

3. Solve the system:

$$\begin{cases} 2x + 3y = 12 \\ x - y = 1 \end{cases}$$

4. A rectangle's length is 3 more than its width. The perimeter is 26. Find the dimensions.

# Algebraic Problem Solving: Functions

## 1 Functions

Functions are mathematical relationships where each input corresponds to exactly one output. Understanding functions is essential for modeling real-world problems and analyzing patterns.

### 1.1 Function Notation and Interpretation

A function is usually written as  $f(x)$ , where  $x$  is the input and  $f(x)$  is the output.

$$f : x \mapsto f(x)$$

**Example:** If  $f(x) = 2x + 3$ , then:

$$f(1) = 2(1) + 3 = 5, \quad f(2) = 2(2) + 3 = 7$$

**Domain and Range:**

- **Domain:** All possible input values ( $x$  values) for the function.
- **Range:** All possible output values ( $f(x)$  values) the function can produce.

### 1.2 Linear Functions and Graphs

A **linear function** has the form:

$$f(x) = mx + b$$

where  $m$  is the slope and  $b$  is the y-intercept.

**Graphing Steps:**

1. Identify the slope  $m$  and y-intercept  $b$ .
2. Plot the y-intercept on the coordinate plane.
3. Use the slope to find a second point.
4. Draw a straight line through the points.

**Example:** Graph  $f(x) = 2x - 1$

- Slope  $m = 2$  (rise/run = 2/1)
- Y-intercept  $b = -1$

### 1.3 Slope and Intercept

**Slope:** Measures the steepness of a line. For two points  $(x_1, y_1)$  and  $(x_2, y_2)$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Y-Intercept:** Point where the line crosses the y-axis ( $x = 0$ ).

**X-Intercept:** Point where the line crosses the x-axis ( $y = 0$ ).

**Example:** Find slope and intercepts of  $y = -\frac{1}{2}x + 3$

- Slope  $m = -\frac{1}{2}$
- Y-intercept:  $(0, 3)$
- X-intercept: Solve  $0 = -\frac{1}{2}x + 3 \implies x = 6$

### 1.4 Exponential Functions (Basics)

Exponential functions have the form:

$$f(x) = a \cdot b^x$$

where  $a \neq 0$ ,  $b > 0$ , and  $b \neq 1$ .

**Key Features:**

- Rapid growth if  $b > 1$ , decay if  $0 < b < 1$
- Y-intercept at  $(0, a)$
- Domain: all real numbers; Range:  $f(x) > 0$  if  $a > 0$

**Example:**  $f(x) = 3 \cdot 2^x$

- $f(0) = 3$ ,  $f(1) = 6$ ,  $f(2) = 12$
- Growth doubles for each increase of 1 in  $x$

## 1.5 Practice Problems

1. Evaluate  $f(x) = 5x - 7$  for  $x = 0, 2, -1$
2. Graph  $f(x) = -x + 4$  and identify slope and intercepts
3. Determine the domain and range of  $f(x) = 3^x$
4. A bacteria population doubles every 3 hours. Write an exponential function for the population if initial count is 100.

# Algebraic Problem Solving: Coordinate Geometry

## 1 Coordinate Geometry

Coordinate geometry, also called analytic geometry, combines algebra and geometry to study points, lines, and shapes on the coordinate plane.

### 1.1 The Coordinate Plane

The coordinate plane consists of two perpendicular number lines:

- **X-axis** – Horizontal axis
- **Y-axis** – Vertical axis

The point where the axes intersect is called the **origin**,  $(0, 0)$ . The plane is divided into four quadrants:

1. Quadrant I:  $(x > 0, y > 0)$
2. Quadrant II:  $(x < 0, y > 0)$
3. Quadrant III:  $(x < 0, y < 0)$
4. Quadrant IV:  $(x > 0, y < 0)$

### 1.2 Plotting Points

A point is represented as  $(x, y)$ , where  $x$  is the horizontal coordinate and  $y$  is the vertical coordinate.

**Example:** Plot the points  $A(3, 2)$ ,  $B(-2, 4)$ ,  $C(-3, -1)$ ,  $D(1, -3)$  in the correct quadrants.

### 1.3 Distance and Midpoint Formulas

**Distance Formula:** The distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Example:** Distance between  $A(1, 2)$  and  $B(4, 6)$ :

$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{9 + 16} = 5$$

**Midpoint Formula:** The midpoint of the segment connecting  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

**Example:** Midpoint of  $A(1, 2)$  and  $B(4, 6)$ :

$$M = \left( \frac{1 + 4}{2}, \frac{2 + 6}{2} \right) = (2.5, 4)$$

## 1.4 Graphing Lines

A line can be represented in slope-intercept form:

$$y = mx + b$$

where  $m$  is the slope and  $b$  is the y-intercept.

**Steps to Graph a Line:**

1. Identify the slope  $m$  and y-intercept  $b$ .
2. Plot the y-intercept on the y-axis.
3. Use the slope to find a second point.
4. Draw a straight line through the points.

**Slope Formula:** For points  $(x_1, y_1)$  and  $(x_2, y_2)$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example:** Graph the line passing through  $(1, 2)$  and  $(3, 6)$ :

$$m = \frac{6 - 2}{3 - 1} = 2 \quad \text{so line equation: } y - 2 = 2(x - 1) \implies y = 2x$$

## 1.5 Practice Problems

1. Plot the points  $(2, 3)$ ,  $(-1, 4)$ ,  $(-2, -2)$ ,  $(3, -1)$  and identify their quadrants.
2. Find the distance and midpoint between  $(0, 0)$  and  $(6, 8)$ .
3. Write the equation of a line with slope  $-1$  passing through  $(2, 3)$  and graph it.
4. Determine the slope of the line connecting  $(-2, 5)$  and  $(4, -1)$ .

# Mathematical Practices in GED Mathematical Reasoning

## 1 Mathematical Practices

Mathematical practices describe the habits of mind, reasoning skills, and tools students use to solve problems and understand quantitative information. Mastery of these practices is essential for success on the GED Mathematical Reasoning test.

### 1.1 Interpreting and Analyzing Quantitative Information

Interpreting quantitative information involves understanding numbers, graphs, tables, and formulas, and drawing accurate conclusions from them. Key skills include:

- Reading data from tables, charts, and graphs.
- Recognizing patterns, trends, and relationships.
- Comparing quantities and calculating percentages, ratios, or rates.
- Identifying relevant information in word problems.

**Example:** A company's sales for January through March are shown in a table. Determine the month with the highest sales and calculate the percentage increase from January to March.

### 1.2 Applying Mathematical Reasoning to Real-World Problems

Real-world problem solving requires connecting abstract mathematical concepts to practical situations. Key skills include:

- Translating verbal problems into equations or expressions.
- Using algebra, geometry, and statistics to solve applied problems.
- Interpreting results in the context of the problem.

**Example:** If a car travels 60 miles in 1.5 hours, determine its average speed in miles per hour.



### 1.3 Constructing Mathematical Arguments

Constructing mathematical arguments involves reasoning logically and communicating solutions clearly. Key skills include:

- Justifying each step in a solution.
- Explaining why a solution is correct or incorrect.
- Using definitions, properties, and theorems to support conclusions.

**Example:** Show why the sum of the angles in any triangle equals  $180^\circ$  using a geometric argument.

### 1.4 Using Appropriate Tools and Technology

Using tools and technology efficiently can enhance problem-solving skills. Common tools include:

- Scientific calculators and graphing calculators.
- Computer software for graphing or data analysis.
- Rulers, protractors, and measuring instruments for geometric problems.
- Online resources for simulations or virtual experiments.

**Example:** Use a graphing calculator to plot  $y = 2x^2 - 3x + 5$  and determine its vertex and axis of symmetry.

### 1.5 Practice Problems

1. A table shows monthly expenses. Calculate the mean, median, and range.
2. A rectangle has a length 3 times its width. If the perimeter is 48 units, find the dimensions.
3. Explain why the diagonals of a rectangle are congruent.
4. Use a calculator to graph  $y = x^3 - 4x$  and identify the x-intercepts.